## Year 11 General Mathematics Worksheet

10 questions on graphs and networks for Year 11 students.


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They're online 3pm-midnight AET, 6 days a week.

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## Questions

1. Which connected graph has:
two nodes (vertices) of degree one, two nodes of degree 2, two nodes of degree 3, one node of degree 4
and two circuits?
a)

b)

c)

d)


Answer: $\qquad$


Which is the correct network for the house plan above?



Answer: $\qquad$
3. Which network has the following adjacency matrix?
A
B
C
D
E $\left[\begin{array}{lllll}\text { A } & \text { C } & D & E \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0\end{array}\right]$


Answer: $\qquad$
5. The cube ABCDKLGH has coloured faces that can be stretched.


The top of the cube is opened out and the elastic faces are stretched out on a flat surface.
The plane graph formed has 8 vertices, 6 faces and 12 edges. This conforms to Euler's Rule, V $+\mathrm{F}-\mathrm{E}=2$

Another composite shape is made of a square pyramid on the right-hand side of a cube.


How many more vertices, $v$, edges, $e$, and faces, $f$, will its graph have than the graph for the cube?
a) $\boldsymbol{v}=\mathbf{1}, \boldsymbol{e}=\mathbf{4}, \boldsymbol{f}=\mathbf{3}$
b) $v=1, e=3, f=4$
c) $v=5, e=4, f=3$
d) $v=5, e=3, f=4$

Answer: $\qquad$
5. An ant starts at the vertex A of a wire frame cube of edge length 25 mm and walks along the edges.


Can the ant visit all of the vertices at least once and return to A by walking less than 225 mm ?

Answer YES or NO.

Answer: $\qquad$

6.

Distances shown are in km.

Ms Sellars, a travelling salesperson, has to go through every town in Sunnyhill Shire at least once.
Jamie Diggars, a road maintenance inspector, needs to travel down every road at least once.

Which one takes the circuit CDEFGHBAC and how long is this path?
a) Diggars, 194 km
b) Diggars, 98 km
c) Sellars, 98 km
d) Cellars, 194 km

Answer:
7. Because there are two vertices of odd degree, you can draw the following graph traversing each arc (edge) only once and without lifting the point of the pencil from the paper.

The circuit is $A B C D B E D A C$.


Is it possible to draw the graph below, traversing each arc only once and without lifting the point of the pencil off the paper?


If the answer is YES, at which point could you start?
a) No
b) Yes, $A$ or $X$
c) Yes, D or J
d) Yes, R or Z

Answer: $\qquad$
8. In a Hamilton Path every node is included exactly once. Hamilton Paths can connect all the vertices of any tetrahedron, cube or octahedron.
Which diagrams show, in green, Hamilton Paths for these solids?


III

a) II, III, VI
b) II, IV, VI
c) I, III, V
d) I, IV, V

Answer: $\qquad$
9. Fiber optic cable is to be connected to eight country towns to provide broadband internet services.
The distances are in kilometres.


What is the shortest length of cable necessary for every town to be on the network?
a) 265 km
b) 645 km
c) 196 km
d) 394 km

Answer: $\qquad$
10. A security night watchman has to visit every building on a school campus four times between 10 pm and 6 am .
It takes him 15 minutes to check a building, and he stops between rounds for 5 minutes for a coffee break.


The matrix shows the times (in minutes) taken to walk between buildings.

|  | AH | Gy | Of | TB | Cl | St |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Assembly <br> Hall | 0 | 3 | 5 | 10 | 6 | 4 |
| Gym | 3 | 0 | 8 | 9 | 10 | 5 |
| Offices <br> Technical <br> block <br> Classrooms | 5 | 8 | 0 | 5 | 7 | 5 |
| Staffroom | 6 | 9 | 5 | 0 | 2 | 4 |
|  | 10 | 5 | 5 | 4 | 3 | 0 |

Which graph shows the quickest route for the security man's round?
a)



Answer: $\qquad$

The Answers.
Hey! No peeking until you've finished...


## Question 1

## Answer: A )



The number of nodes is:
$2($ of degree 1$)+2($ of degree 2$)+2($ of degree 3$)+1$ (of degree 4$)$
= 7 nodes.
Every edge connects two nodes.
To find the number of edges find half the sum of all the degrees.
The number of edges is $(2 \times 1+2 \times 2+2 \times 3+1 \times 4) / 2=8$ edges.
Eliminate the answers that do not have 7 nodes and 8 edges.
Select the remaining answer that has two circuits.
A circuit is a path that can start and end at the same node.

## Question 2

## Answer: B )

Outside

Both Bathrooms and Bedrooms 2 and 3 all have degree 1. The Living Room and the Main Bedroom both have degree 2.

The Dining Room and the Kitchen both have degree 3.
The Gallery, Outside and the Family Room all have degree 4.
The Hall has degree 6.
The total number of edges should be 16.

## Question 3

Answer: C )


The 1 in the first row and second column tells you that there is an edge joining node A and node B and that you can go from A to B.
The 0 in the second row and first column tells you that you cannot go from $B$ to A.

There is 1 in the second row and fourth column. This tells you that you can go from B to D. Eliminate any graph that does not have a directed link from B to D. There is 1 in the third row and fourth column. This tells you that you can go from C to D . Eliminate any graph that does not have a directed link from C to D .

There is 1 in the fifth row and first column. This tells you that you can go from E to A. Eliminate any graph that does not have a directed link from E to A The Os mean that there is no directed single edge between the nodes. although there may be a path made of two or more edges.

## Question 4

Answer: A ) v = 1, e = 4, f=3.

Since Euler's Rule will still hold true $v+f-e=0$.
Using this to check the four choices given in the answers, only one of them is possible, which is $v=1, e=4, f=3$.

The one extra vertex comes from the point (apex) of the pyramid.
The four extra edges are the slanting edges of the pyramid.
The four slanting faces of the pyramid have to be added but one face of the cube has been covered leaving a net increase of three faces.

## Question 5

Answer: Yes.

If the ant goes from $A$ to $B$ to $C$ to $D$ it will have walked 75 mm and visited all the top vertices.
It can now go 25 mm down to H on the bottom level.
From H it can go to G to F to E , a further 75 mm .
The ant has now visited all 8 vertices and needs only to go 25 mm up to A above E .
In total the ant will have walked 200 mm (<225 mm).
Its route was ABCDHGFEA.
Other routes are possible.

## Question 6

Answer: C ) Sellars, 98 km
Check that every town has been circled.
If so, Ms Sellars could take this route.
Add the distances $(5+11+18+16+14+12+10+14) \mathrm{km}$ as you pass along the edges.
This is called a Postman's Circuit.
Now look at the edges that you have coloured.
If every edge is not coloured, Jamie Diggars did not travel on this route.

## Question 7

Answer: C ) Yes, D or J
P, H, R, Z are of degree 2. These can only be visited once.
$A$ and $X$ are of degree 4 . These can be visited twice.
D and J are of degree 5 . These can be visited twice as well one being the starting point and the other being the finishing point.

A possible circuit is D PXHJXDJARDAZJ.
Other routes are possible.
The answer is YES and you can start at either D or J.

## Question 8

Answer: A ) II, III and VI

If the whole path is traversed and a node has a degree greater than 2 , the node will be visited at least twice.
These paths are not circuits. The finishing node is not the same as the starting node. The starting and finishing nodes have degree one.

In the Hamilton Paths no node has degree greater than two.
I, IV and V are not Hamilton Paths.
II, III and VI are Hamilton Paths.

## Question 9

Answer: A ) 265 km


The shortest cable length is 265 km .

## Question 10

## Answer: B )



The smallest number, 2, is in the Technical Block column. Highlight this whole column.
The highlighted 2 is in the Classrooms row. Draw a vertical line down the Classrooms column. Join the dots for the Technical Block and the Classrooms on your map (2 Minutes).

Highlight the smallest number, 3 , in the Classrooms row that is not crossed out or highlighted. This is in Staffroom column. Draw a vertical line down the Staffroom column. Join the Classrooms and the Staffroom dots on your map (3 minutes).
Highlight the smallest number, 4, in the Staffroom row that is not crossed out or already highlighted. This is in the Assembly Hall column. Draw a vertical down Assembly Hall column. Join the Staffroom and the Assembly Hall dots on your map (4 minutes).
Highlight the smallest number, 3 , in the Assembly Hall row that is not crossed out or already highlighted. This is in the Gymnasium column. Draw a vertical line down the Gymnasium column. Join the Assembly Hall and the Gymnasium dots on your map (3 minutes).

To complete the circuit, join the Gymnasium to the Offices (8 minutes) and the Offices to the Technical Block (5 minutes).

Shortest total walking time $=25$ minutes.
NOTE: Total inspection time + coffee break = 95 minutes.

Total circuit time $=25$ minutes +95 minutes $=2$ hours.
The security guard can start at any location - most likely the Offices - and complete four circuits in the 8 hours between 10 pm and 6 am .

