PERMUTATIONS and COMBINATIONS

If the order doesn't matter, it is a **Combination**.
If the order **does** matter it is a **Permutation**.

PRACTICE!

Determine whether each of the following situations is a Combination or Permutation.

1. Creating an access code for a computer site using any 8 alphabet letters.

2. Determining how many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from.

3. Voting to allow 10 new members to join a club when there are 25 that would like to join.

4. Finding different ways to arrange a line-up for batters on a baseball team.

5. Choosing 3 toppings for a pizza if there are 9 choices.

Answers: 1. P 2. P 3. C 4. P 5. C

Combinations:

Suppose that you can invite 3 friends to go with you to a concert. If you choose Jay, Ted, and Ken, then this is no different from choosing Ted, Ken, and Jay. The order that you choose the three names of your friends is not important. Hence, this is a Combination problem.

Example Problem for Combination:

Suppose that you can invite 3 friends to go with you to a concert. You have 5 friends that want to go, so you decide to write the 5 names on slips of paper and place them in a bowl. Then you randomly choose 3 names from the bowl. If the five people are Jay, Ted, Cal, Bob, and Ken, then write down all the possible ways that you could choose a group of 3 people.

Here are all of the possible combinations of 3:

Jay, Ted, Cal	Ted, Cal, Bob	Cal, Bob, Ken
Jay, Ted, Bob	Ted, Cal, Ken	
Jay, Ted, Ken	Ted, Bob, Ken	

Jay, Cal, Bob
Jay, Cal, Ken

Jay, Bob, Ken

To determine the number of combinations we use the rule:

$${}_nC_r = \frac{n!}{(n-r)!\ r!}$$

Where n represents the total number of things to choose from and r represents the number of things to be selected.

In the previous problem, n = 5 and r = 3.

$${}_5C_3 = \frac{5!}{(5-3)!3!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{(2\cdot1)\cdot3\cdot2\cdot1} = 10$$

<u>PRACTICE:</u> Go back to the beginning of this lesson and determine the number of possible combinations for #3 and #5. Answers: 3,268,760 and 84 , respectively.

Permutations:

A permutation is used when re-arranging the elements of the set creates a new situation.

Example Problem for Permutation:

How many ways could we get 1^{st}, 2^{nd}, and 3^{rd} place winners in a race with the following 4 people? Jay, Sue, Kim, and Bob

**Note: since winning first place is different than winning second place, the set {Jay, Sue, Kim} would mean something different than {Jay, Kim, Sue}.

Here is why they are different:
{Jay, Sue, Kim} indicates that Jay won 1^{st} , Sue won 2^{nd}, and Kim won 3^{rd} .
{Jay, Kim, Sue} indicates that Jay won 1^{st}, Kim won 2^{nd}, and Sue won 3^{rd}

Here are all of the possibilities for 1^{st}, 2^{nd}, 3^{rd} in that order:

Jay, Sue, Kim	Sue, Kim, Bob	Kim, Sue, Bob	Bob, Jay, Sue
Jay, Kim, Sue	Sue, Bob, Kim	Kim, Bob, Sue	Bob, Sue, Jay
Jay, Sue, Bob,	Sue, Jay, Kim	Kim, Jay, Sue	Bob, Jay, Kim
Jay, Bob, Sue	Sue, Kim, Jay	Kim, Sue, Jay	Bob, Kim, Jay
Jay, Kim, Bob	Sue, Jay, Bob	Kim, Jay, Bob	Bob, Sue, Kim
Jay, Bob, Kim	Sue, Bob, Jay	Kim, Bob, Jay	Bob, Kim, Sue

There are 24 permutations.

Another way to determine the number of permutations is to use the following formula:

$_nP_r = \frac{n!}{(n-r)!}$ Where n represents the total number of things to choose from and r represents the number of things to be selected.

In the previous problem, $_4P_3 = \frac{4!}{(4-3)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1!} = 24$

PRACTICE: Go back to the beginning of this lesson and determine the number of possible permutations for #1 and #2. Answers: $6.299 * 10^{10}$ and 90 , respectively.

MORE COMBINATION AND PERMUTATION PRACTICE PROBLEMS:

1. Suppose that 7 people enter a swim meet. Assuming that there are no ties, in how many ways could the gold, silver, and bronze medals be awarded?

2. How many different committees of 3 people can be chosen to work on a special project from a group of 9 people?

3. A coach must choose how to line up his five starters from a team of 12 players. How many different ways can the coach choose the starters?

4. John bought a machine to make fresh juice. He has five different fruits: strawberries, oranges, apples, pineapples, and lemons. If he only uses two fruits, how many different juice drinks can John make?

5. How many different four-letter passwords can be created for a software access if no letter can be used more than once?

6. How many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from.

7. There are 25 people who work in an office together. Five of these people are selected to go together to the same conference in Orlando, Florida. How many ways can they choose this team of five people to go to the conference?

8. There are 25 people who work in an office together. Five of these people are selected to attend five different conferences. The first person selected will go to a conference in Hawaii, the second will go to New York, the third will go to San Diego, the fourth will go to Atlanta, and the fifth will go to Nashville. How many such selections are possible?

9. John couldn't recall the Serial number on his expensive bicycle. He remembered that there were 6 different digits, none used more than once, but couldn't remember what digits were used. He decided to write down all of the possible 6 digit numbers . How many different possibilities will he have to create?

10. How many different 7-card hands can be chosen from a standard 52-card deck?

11. One hundred twelve people bought raffle tickets to enter a random drawing for three prizes. How many ways can three names be drawn for first prize, second prize, and third prize?

12. A disc jockey has to choose three songs for the last few minutes of his evening show. If there are nine songs that he feels are appropriate for that time slot, then how many ways can he choose and arrange to play three of those nine songs?

Answers:

1. ${}_{7}P_{3}$ = 210

2. ${}_{9}C_{3}$ = 84

3. ${}_{12}P_{5}$ = 95,040

4. ${}_{5}C_{2}$ = 10

5. ${}_{26}P_{4}$ = 358,800

6. ${}_{10}P_{2}$ = 90

7. ${}_{25}C_{5}$ = 53,130

8. ${}_{25}P_{5}$ = 6,375,600

9. ${}_{10}P_{6}$ = 151,200

10. ${}_{52}C_{7}$ = 133,784,560

11. ${}_{112}P_{3}$ = 1,367,520

12. ${}_{9}P_{3}$ = 504